Quantifying information transmitted by a single stimulus.

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Abstract

Information theory [1] provides a natural mathematical framework to answer to the question *how much* information is contained in the neural patterns. Usually, in an experiment we choose a controlled set of stimuli $\mathcal{S}$, and we record the elicited neural responses $r \in \mathcal{R}$ when one stimulus $s \in \mathcal{S}$ is repeatedly presented with a known a priori probability $p(s)$. Once we have collected all the data, we can estimate the corresponding conditional probability $p(r|s)$ and the probability distribution of responses averaged over the stimuli $p(r)$; then compute the mutual information

$$I(\mathcal{S};\mathcal{R}) = \sum_{s\in\mathcal{S}}\sum_{r\in\mathcal{R}} p(s)p(r|s)\ \log_2\left[\frac{p(r|s)}{p(r)}\right], \tag{1}$$

or, introducing the entropy of a probability distribution: $H(\mathcal{R}) = \sum_{r\in\mathcal{R}} p(r)\log_2 p(r)$,

$$I(\mathcal{S};\mathcal{R}) = H(\mathcal{R}) - H(\mathcal{R}|\mathcal{S}) \tag{2}$$

where $H(\mathcal{R}|s) \equiv \sum_{s\in\mathcal{S}} p(s)H(\mathcal{R}|s)$ is the conditional entropy.

Mutual information summarizes how much we can tell about the stimuli reading the neural response (or vice-versa).

Thus, Shannon mutual information provides a quantitative measure of the averaged information transmitted by a set of responses about a set of stimuli (or vice-versa). In many cases it is interesting to know which the stimulus is that contained more information in a given set, or, similarly, which the more informative response is. Furthermore, a quantitative estimation of the information transmitted by a single symbol (stimulus or response) can be used to study the different encoding capability of the system, such as two different stimuli are encoded independently, or not. In his original formulation, Shannon did not provide any insights about how much information can be carried by a single symbol. After Shannon seminal work, many definitions of one-symbol specific information have been introduced [2]. To author knowledge, all of them have been proposed in the framework of neural response analysis, and, in particular, for investigating information carried by a single stimulus, so they are usually referred as *stimulus specific information.*

Two possible definitions of such quantity can be immediately inferred from Eq. 1 and Eq. 2, simply taking the single stimulus contribution to the sum:

$$\begin{aligned} I_1(s) &= \sum_{r\in\mathcal{R}} p(r|s)\log_2\frac{p(r|s)}{p(r)} \\ I_2(s) &= H(\mathcal{R}) - H(\mathcal{R}|s) \end{aligned}$$

These two quantities assume in general different values, although they both clearly average to mutual information, i.e. $\sum_{s \in \mathcal{S}} p(s) I_{1,2}(s) = I(\mathcal{S};\mathcal{R})$. Note that any weighted average of them has also this property, and it can represent a possible definition of stimulus specific information. So, we have an infinite number of possible choices for a stimulus-dependendent decomposition of mutual information. In addition, recently, two other definitions have been proposed: local information [3] and SSI [4], and applied to the study of encoding of spatial position and sensory stimuli, respectively.

In this study we have analysed in details the features of these four quantities and compare their different meanings in different contexts. We focused on some less understood properties, such as additivity and single stimulus information independence. We have also found new relationships between these quantities, showing, for example, that surprise and local information coincide in the limit of very unlikely (*surprising*) stimuli.

To better illustrate the behavior of these stimulus specific informations in a realistic case, we have considered the case of location coding in hippocampus. In this case, we can easily show how these quantities are sensitive to different features of neural response. For example, place field results the optimal (more informative) stimulus using one I_2, and the less informative one according to I_1.

In conclusions, there is no “fully” satisfactory definition of this stimulus specific information. In the sense that none of these definitions shares the mathematical properties of Shannon mutual information, such as positive definite and additivity, which make it so appealing from the application point of view. But, each of these measures may be used to investigate different aspects of single stimulus contribution to information transmission.

References

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