INPUT-OUTPUT RELATIONS IN BINDING NEURON

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ABSTRACT

The binding neuron model is inspired by numerical simulation of Hodgkin-Huxley-type point neuron stimulated from many synaptic inputs [1], as well as by the leaky integrate-and-fire model [2]. It is expected that input stream in any synapse is poissonian. In this case, from mathematical point of view, all inputs can be replaced with a single one with poissonian stream in it having its intensity equal to the sum of all intensities in the synapses (Fig.1). The binding neuron works as follows. Any input impulse is stored in the neuron during time τ and then forgotten. If the number of stored impulses, Σ, becomes equal to the threshold one, N_0, the neuron sends an output impulse, clears its internal memory and is ready to receive impulses from the input stream. No refraction is expected.

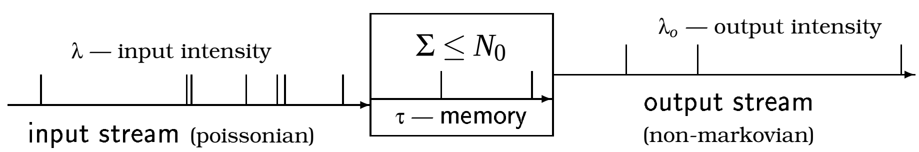

Fig.1: Binding neuron

The leaky integrate-and-fire neuron [2] becomes the binding one, if we put the refraction time equal to zero and choose the EPSP time course being the rectangular of width τ and height 1, instead of the exponential decay.

Our purpose is to characterise the output stream of binding neuron in terms of λ, τ, N_0. The output stream will be a stationary stochastic process and as such, can be characterized by its intensity, λ_o, which is defined as mean number of events per unit time [3]. For $N_0 = 2$ and 3 we obtain the neuronal transfer function which gives λ_o as a function of λ:

$$\lambda_o = \frac{1 - e^{-\lambda\tau}}{2 - e^{-\lambda\tau}}\lambda, \qquad N_0 = 2,$$

$$\lambda_o = \frac{1 - e^{-\lambda\tau} - e^{-\lambda\tau} S(\lambda\tau)}{2 - e^{-\lambda\tau} + (1 - e^{-\lambda\tau}) S(\lambda\tau)}\lambda, \qquad N_0 = 3.$$

Here for $q \leq \log(4)$

$$S(q) = \frac{s(q)\sin\left(\frac{q e^{-q/2} s(q)}{2}\right) + \left(e^{\frac{q}{2}} - 2e^{-\frac{q}{2}}\right)\cos\left(\frac{q e^{-q/2} s(q)}{2}\right) + 1}{2e^{-\frac{q}{2}}\cos\left(\frac{q e^{-q/2} s(q)}{2}\right) + 1},$$

where $s(q) = \sqrt{4 - e^q}$.

And for $q \geq \log(4)$

$$S(q) = \frac{-s_1(q)\sinh\left(\frac{q e^{-q/2} s_1(q)}{2}\right) + \left(e^{\frac{q}{2}} - 2e^{-\frac{q}{2}}\right)\cosh\left(\frac{q e^{-q/2} s_1(q)}{2}\right) + 1}{2e^{-\frac{q}{2}}\cosh\left(\frac{q e^{-q/2} s_1(q)}{2}\right) + 1},$$

where $s_1(q) = \sqrt{e^q - 4}$.

For $N_0 = 2$ the interspike interval probability density distribution for the output stochastic process, $p(t)\,dt$, is obtained in closed form. The $p(t)$ is unimodular with $p(0) = 0$ and maximum at $t = \min(\tau, \frac{1}{\lambda})$.

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Keywords: binding neuron, poissonian stream, threshold, transfer function

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